

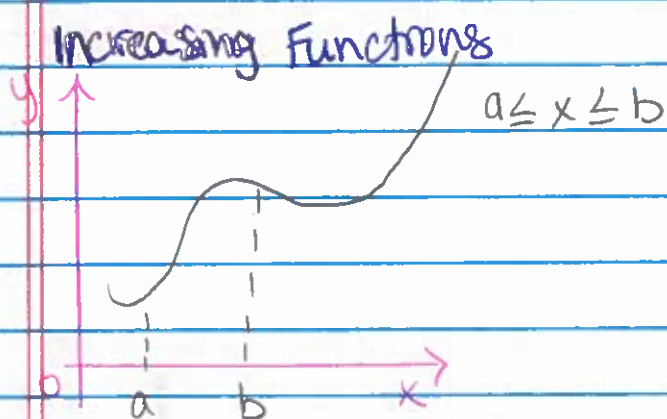
Ch 5 p1

MATH

June 18

Applications of the Derivative

- First Derivative test
- Increasing and decreasing functions



The tangent lines have positive slopes

Since $\frac{dy}{dx} = f'(x)$ is the slope function.

If $y = f(x)$ increases on an interval $a \leq x \leq b$, then its derivative $\frac{dy}{dx} = f'(x)$ is positive.

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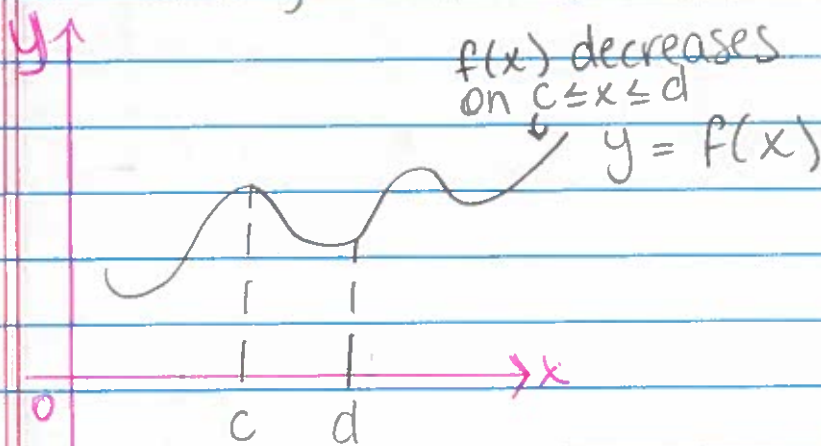
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We also notice that when a function increases on $a \leq x \leq b$, the tangent lines on this interval have become positive slopes. Since $\frac{dy}{dx} = f'(x)$ is the slope function, we conclude that if $y = f(x)$ increases on an interval $a \leq x \leq b$, then its derivative $\frac{dy}{dx} = f'(x)$ is positive on this interval.

Decreasing functions

If a function $y = f(x)$ behaves such that $f(x_i) > f(x_{i+1})$ where $x_i < x_{i+1}$ for all $c \leq x \leq d$, then the function is said to be decreasing on this interval.



We also note that when a function decreases on $c \leq x \leq d$, the tangent lines on this interval have negative slopes. We conclude that if $y = f(x)$ decreases on interval $c \leq x \leq d$, then its derivative is negative on this interval.

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Critical points

Critical points for a function $y=f(x)$ occur where

① $\frac{dy}{dx} = 0$ OR

② $\frac{dy}{dx} = \text{DNE}$ (does not exist)

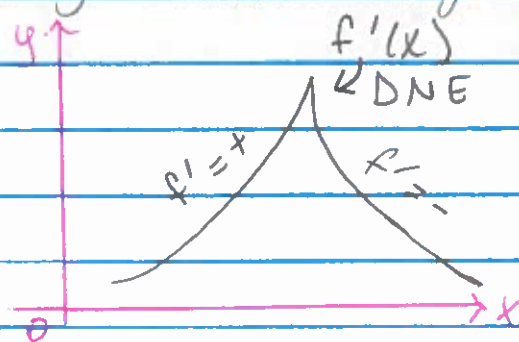
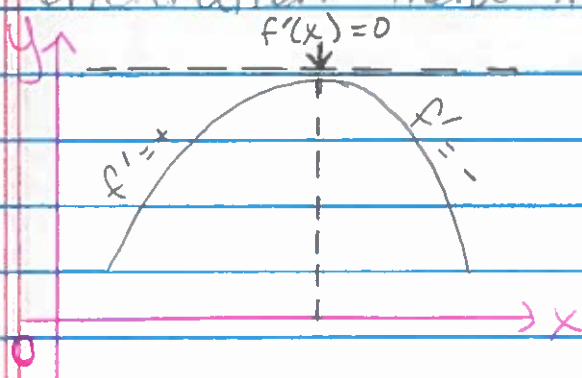
provided this point is on the domain of $y=f(x)$.

Critical points are where the function may change orientation from decreasing to increasing or increasing to decreasing

Sometimes a function may not change orientation at critical points.

Relative Extreme Values

- ① A critical point where the function changes orientation from increasing to decreasing



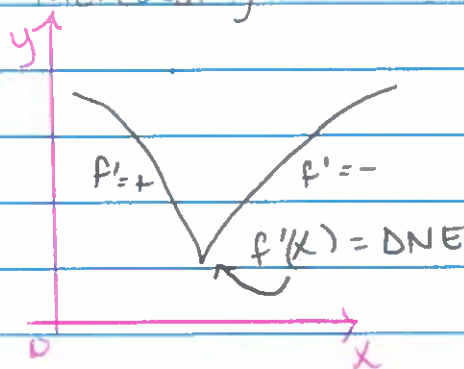
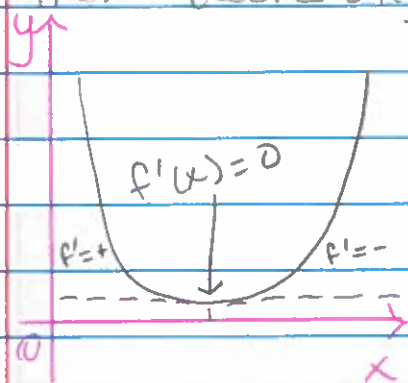
We have a relative maximum at the critical point where $\frac{dy}{dx} = f'(x)$ changes sign from positive to negative.

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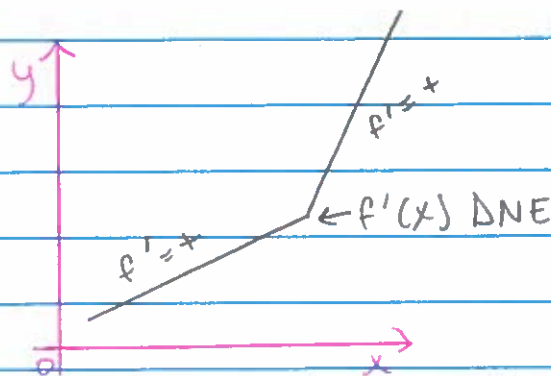
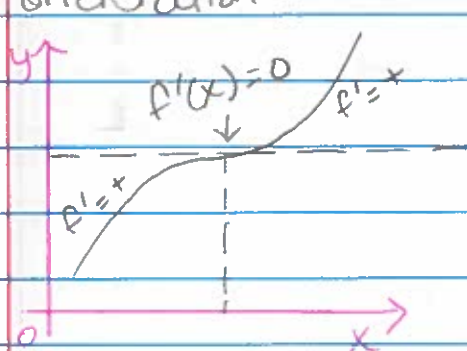
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- ② A critical point where the function changes from decreasing to increasing



we have a relative minimum at the critical point where $\frac{dy}{dx} = f'(x)$ changes sign from negative to positive

- ③ A critical point where the function does not change orientation



This critical point is neither a relative maximum nor a relative minimum if $f'(x)$ does not change sign.

we conclude the definition of the First Derivative Test.

Step 0: Determine the domain of $y=f(x)$

Step 1: Find its first derivative, $\frac{dy}{dx} = f'(x)$

Step 2: Find the Critical points i) $\frac{dy}{dx} = 0$

ii) $\frac{dy}{dx} = f'(x) DNE$

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only points on the domain are acceptable as critical points.

Step 3: perform a sign analysis for $\frac{dy}{dx} = f'(x)$ based on critical points on the domain.

Conclusion step:

↳ if $\frac{dy}{dx} = f'(x)$ changes sign from positive to negative on a critical point, then this critical point is a relative maximum.

↳ if $\frac{dy}{dx} = f'(x)$ changes sign from negative to positive on a critical point, then this critical point is a relative minimum.

↳ if $\frac{dy}{dx} = f'(x)$ does not change sign on a critical point, then this critical point is neither a relative maximum nor a relative minimum.

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$$y = -4x^2 - 7x + 5 \text{ on } (-\infty, \infty)$$

Its first derivative is

$$\frac{dy}{dx} = -8x - 7$$

critical points occur when

$$(1) \frac{dy}{dx} = 0$$

$$\Rightarrow -8x - 7 = 0$$

$\Rightarrow x = -\frac{7}{8}$ ← acceptable as a critical point since it is on the domain

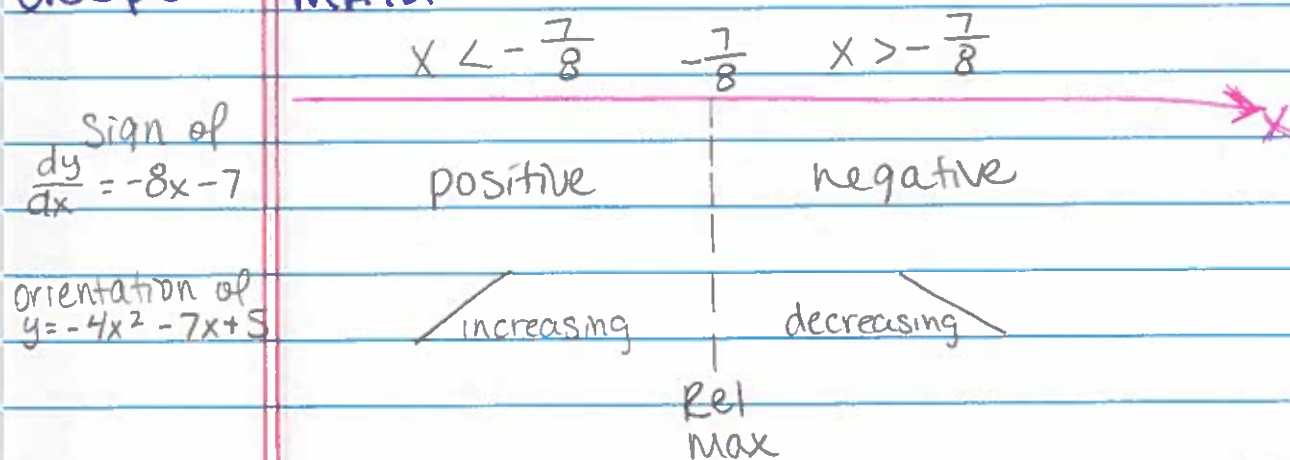
(2) $\frac{dx}{dy}$ DNE, but $\frac{dy}{dx} = -8x - 7$ is a polynomial and therefore exists everywhere on the domain.

So this criteria does not yield any critical point.

Unsp6

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∴ the function $y = -4x^2 - 7x + 5$ increases on $x < -\frac{7}{8}$ and decreases on $x > -\frac{7}{8}$. It has a relative maximum at $x = -\frac{7}{8}$.

Function value at $x = -\frac{7}{8}$,

$$\begin{aligned}
 y &= -4\left(-\frac{7}{8}\right)^2 - 7\left(-\frac{7}{8}\right) + 5 \\
 &= -4\left(\frac{49}{64}\right) + \frac{49}{8} + 5 \\
 &= -\frac{49}{16} + \frac{98}{16} + \frac{80}{16} \\
 &= \frac{129}{16}
 \end{aligned}$$

Relative maximum profit is at $\left(-\frac{7}{8}, \frac{129}{16}\right)$.

19p139

$$y = \ln x^2 - 6x \quad \text{on } (-\infty, 0) \cup (0, \infty)$$

$$\Rightarrow y = 2 \ln x - 6x$$

$$\Rightarrow \frac{dy}{dx} = 2\left(\frac{1}{x}\right) - 6$$

$$= \frac{2}{x} - \frac{6x}{x} = \frac{2-6x}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2(1-3x)}{x}$$

$$\Rightarrow \frac{2(1-3x)}{x} = 0$$

$$\Rightarrow 2(1-3x) = 0$$

$$\Rightarrow 1-3x = 0$$

$\Rightarrow x = \frac{1}{3}$ ← acceptable as a critical point since it is on the domain.

① critical points occur when i) $\frac{dy}{dx} = 0$

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② $\frac{dy}{dx}$ DNE

$\Rightarrow \frac{2(1-3x)}{x}$ DNE

$\Rightarrow x=0$ $\leftarrow$ not acceptable as a critical point since it is not on the domain

$x < 0$ $0 < x < \frac{1}{3}$ $\frac{1}{3}$ $x > \frac{1}{3}$

Sign of 2	positive	positive	positive
Sign of $1-3x$	positive	positive	negative
Sign of x	negative	positive	positive
Sign of $\frac{dy}{dx} = \frac{2(1-3x)}{x}$	negative	positive	negative
orientation of $y = 1.5x^2 - 6x$	decreasing	increasing	decreasing
	NOT ON DOMAIN	Rel max	

(3.6) Consumption Function

When we earn an income, I , we can either spend it, K , or we can save it, S .

$$I = K + S$$

$$\Rightarrow \frac{dI}{dI} = \frac{dK}{dI} + \frac{dS}{dI}$$

$$\Rightarrow \frac{dK}{dI} + \frac{dS}{dI} = 1$$

where $\frac{dK}{dI}$ = marginal propensity to consume

and $\frac{dS}{dI}$ = marginal propensity to save

BOTH $0 \leq \frac{dK}{dI} \leq 1$ and $0 \leq \frac{dS}{dI} \leq 1$

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ex sp 98

given $K(I) = 0.96I - 0.005I^2$ on $12 \leq I \leq 55$
when I is in thousands.

(i) marginal Propensity to Consume

$$\frac{dK}{dI} = 0.96 - 0.01I$$

(ii) given Brock's current income of $I = 52$ (I is in thousands)

Brock's propensity to save is

$$\frac{dK}{dI} = 0.96 - 0.01(52)$$

$$= 0.44 = 44\% \text{ of any income he earns.}$$

If Brock wins \$4000 at a casino, he is likely to spend 44% of it, or

$$(\$4000)(0.44) = \$1760.$$

He is likely to save $\$4000 - \$1760 = \$2240$.

$$\text{OR } \frac{dK}{dI} + \frac{dS}{dI} = 1$$

$$\Rightarrow \frac{dS}{dI} = 1 - \frac{dK}{dI}$$

$$= 1 - 0.44 = 0.56$$

Brock is likely to save $(\$4000)(0.56) = \2240